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LETTER TO THE EDITOR

Non-universality in the dynamics of the one-dimensional Potts model

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Abstract. We discuss two recent papers on the dynamics of the one-dimensional Potts model which yield different values of the dynamical critical exponent. We show that in fact there are choices of the transition rates which yield arbitrarily large non-universal exponents.

The possible universality classes for the one-dimensional kinetic Ising model have been extensively treated in the recent literature (Deker and Haake 1979, Haake and Thol 1980, Kimball 1979, Cordery *et al* 1981). Haake and Thol introduced a choice of transition rates for which the exponent z varies continuously from 2 up to 4.

The case of the dynamics of the one-dimensional q-state Potts model has only recently received attention. In a recent paper (Weir and Kosterlitz 1986) we found the value $z=2$ for the dynamical critical exponent for the Potts model with simple relaxational dynamics (model A, Hohenberg and Halperin 1977). However another author (Lage 1985a, b) found the value $z=3$ with a different choice of the transition rates.

We would like to clarify the situation by constructing a more general transition rate that satisfies detailed balance, and allows one to continuously vary the exponent. While our rate is not the most general possible, it includes the choice of Lage as a special case. Whereas Lage allows the transition rate to depend only on the initial state of the system, we also include information on the final state. For the Ising case, our rate reduces to that introduced by Glauber (1963), and so has the exponent $z=2$. We are thus considering a different generalisation of the Glauber rate to that introduced by Haake and Thol.

We use the method developed by Cordery *et al* (1981) in which the exponent z is determined by calculating the rate at which a domain wall diffuses at low temperature. They describe the system near the transition in terms of large blocks of size ξ, the correlation length, and regard the relaxation time τ as the time required to flip such a block by the motion of a domain wall a distance ξ.

We calculate the flip rate w for a spin at the boundary between two regions of different Potts states. If this has a dependence on the correlation length of the form ξ^{-x}, then the exponent z is given by the usual random walk arguments, as $z=2+x$.

We define the transition rates

$$W_{s_i\to\hat{s}_i}=\frac{\exp(\beta\mu H(s_i)-\beta\lambda H(\hat{s}_i))}{\mathrm{Tr}_{s_i\neq\hat{s}_i}\,[\mathrm{Tr}\exp(\beta\mu H(s_i)-\beta\lambda H(\hat{s}_i))]}\tag{1}$$

which satisfy detailed balance provided $\lambda + \mu = 1$. The s_i and $\hat{s}_i$ denote the initial and final states at the site i. Note that the μ and λ describe the relative importance given to the initial and final states at the site i. The case $\mu = 1$, $\lambda = 0$, corresponds, up to a factor, to that considered by Lage; thus he finds a flip rate independent of the final state. In the $q = 2$ case, as mentioned above, equation (1) becomes independent of μ and λ, and reduces to the Glauber choice (Glauber 1963).

We find the rate for a spin flip at a domain wall to be given by

$$W^{-1} = 2 + 2(q-2)[\exp(\mu\beta J) + \exp(-\lambda\beta J)] + (q-2)(q-3)\exp[(\mu-\lambda)\beta J] \tag{2}$$

which yields exponents

$$\begin{array}{lll} q = 2 \text{ (Ising)} & z = 2 & \\ q = 3 & z = 2 & (\mu < 0) \\ & z = 2 + \mu & (\mu > 0) \\ q > 3 & z = 2 & (\mu < 0) \\ & z = 2 + \mu & (0 < \mu < 1) \\ & z = 1 + 2\mu & (\mu > 1). \end{array} \tag{3}$$

We have also carried out a lower-bound calculation for the case $\mu = 0$ using the same variational method as Lage (1985a). This yields the lower bound $z \geqslant 2$ for $\mu = 0$, in contrast to the lower bound $z \geqslant 3$ which Lage derived for $\mu = 1$. This may be taken as further evidence that the form given above for the exponents is correct. In fact, a renormalisation group calculation for the case $\mu = 1$ (Lage 1985b) showed that $z = 3$; that is the lower bound is actually attained.

In the case of the Ising model, with the flip rate introduced by Haake and Thol (1980), Cordery *et al* showed that if the exponent calculated from the motion of the domain wall by single steps is too slow, it might be faster to first flip the spin one lattice spacing away, and then the spin at the boundary. We find that with our choice of rates, this process always yields a larger exponent, and so does not provide the upper limit $z \leqslant 4$ that it does in the Ising case with Haake and Thol's choice of rates. This apparent non-universality is different from that caused by the simple inclusion of an overall factor, which depends on the correlation length, in all of the flip rates. This factor can be absorbed into a redefinition of the timescale, and can be adjusted to give one any exponent one wants.

The essential point is that, at low temperatures, only the lowest energy excitations above the ground state survive, the states consisting of widely separated domain walls. This state relaxes to equilibrium by the motion of the domain walls, so this is the slowest relaxing mode. We really ought to define our timescale so that the microscopic flip rates describing the motion of the domain walls are of order unity. This will obviously imply that $z = 2$.

In the case of the Ising model, it is sufficient (Achiam 1983) to demand that the part of the flip rate which does not depend on s_i or $\hat{s}_i$ has no functional dependence on the macroscopic correlation length. This is because in the Ising case, this factor is the only one responsible for the domain wall slowing which increases z. At a domain boundary $\sigma_{i-1} = -\sigma_{i+1}$, and so $W_{\sigma_i \to -\sigma_i} = W_{-\sigma_i \to \sigma_i}$, thus the rates for domain motion depend only on the neighbours and not on the spin at site i. This is no longer true in the Potts case, since transitions to many other states are possible. Thus correlation length factors can come from any part of the flip rate, not just the part which does not depend on s_i and $\hat{s}_i$. The choice of rates of Haake and Thol for the Ising model which yield a non-universal exponent is forbidden by the restrictions mentioned above. The choice of rates used by Achiam and Kosterlitz (1978), which is simply an exponential of half the energy difference and was the rate used in our study of the Potts model as well as that of Forgacs *et al* (1980), does not suffer from the drawbacks mentioned above and gives $z = 2$ for all q. One should be careful in any calculation of the exponent z to carefully specify one's choice of rates. Leyvraz and Jan (1986) claim $z = 2$ for the three- and four-state Potts model, but do not give any details of their calculation.

In summary, Lage's choice of rates, which as has been noted is a special case of the general rate introduced above, is simply one of many possible non-universal choices of the rates. One ought to either impose no restrictions (other than detailed balance) on the transition rates for the dynamics one imposes on a model, giving a whole range of exponents, or demand that the domain wall rates be of order unity (which eliminates the non-universality) and gives $z = 2$. These difficulties in the definition of the rates should disappear if one is looking at a model with a finite critical temperature.

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